Fermi problems

You don’t need a calculator or computer to get a feel for the timescales and ‘sizes’ of geological processes. You can gain a more instinctive understanding of ‘how the Earth works’ by doing some ‘back of the envelope’ reasoning.

This quantitative reasoning approach is widely used by physicists. The approach was first popularised by Enrico Fermi, so some people call them Fermi Problems. You can use the approach at all levels, most usefully as a ‘common sense’ checker.

Rules of engagement

Your envelope is small, so follow these rules:

- Use scientific notation (× 10^n) to represent numbers.
- Never carry more than 2 significant figures across.
- Use approximations where possible, to simplify things.
- Be confident in estimating typical time- and length-scales. You’ll soon get a sense of whether you are close.

Practice question - timescales

The standard S. I. unit for time is the second, but many geological processes deal with longer timescales:

- thousands (× 10^3) of years (ka or kyr)
- millions (× 10^6) of years (Ma or Myr)
- billions (× 10^9) of years (Ga or Byr)

Work out simple approximations for the following, in seconds:
(a) 1 day    (b) 1 week    (c) 1 year

What to do

Think about your question (problem A, below). Try to solve it using quantitative reasoning – use the back of a real envelope if you like! If you get stuck, ask your teacher for a solution sheet. Work through the given answer until you are sure you understand the method used. Then try using a slightly different approach to answer the same question.

Next plan how to present your problem to the rest of your group. You will need to consider the questions below:
- What clues will you give if they get stuck?
- Can you think of a similar problem, to give them more practice using the technique in a similar context?
- How will you go through your solution to the problem if anyone is completely stuck?

Your question (problem A)

How long does it take for the sinking slab (subducting plate) in a subduction zone to reach the volcanic front?

Useful data and other information

In the animation we see Ossie speeding up time, to travel along the descending slab, until he reaches the point beneath Stromboli where fluid escapes from the slab and starts to rise.

- In a ‘typical’ subduction zone the depth from the volcano to the top of the Benioff zone = 100 km. The Benioff zone is the boundary between two plates where one moves beneath the other.
- A typical convergence rate in a subduction zone is 5 cm/yr (50 km/Ma).
- A typical slab dip is 45°.
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- billions (× 10^9) of years (Ga or Byr)

Work out simple approximations for the following, in seconds:

(b) 1 day  (b) 1 week  (c) 1 year

What to do

Think about your question (problem B, below). Try to solve it using quantitative reasoning – use the back of a real envelope if you like! If you get stuck, ask your teacher for a solution sheet. Work through the given answer until you are sure you understand the method used. Then try using a slightly different approach to answer the same question.

Next plan how to present your problem to the rest of your group. You will need to consider the questions below:

- What clues will you give if they get stuck?
- Can you think of a similar problem, to give them more practice using the technique in a similar context?
- How will you go through your solution to the problem if anyone is completely stuck?

Your question (problem B)

How long does it take for magma to rise from above the plate in the subduction zone to reach the volcano and erupt?

Useful data and other information

The answer to this question is not well known at all! Use a thought experiment to answer the question. Assume the following:

- Magma rises through the mantle in blobs, like those in a lava lamp. It rises 100 km from above the Benioff zone (the boundary between the subducting plate and the plate above it) at 5 mm/yr.
- The magma then rises 80 km in the cracks of the overlying plate at 1 mm/second.
- In an eruption magma rises 10 km through a conduit at 1 m/second.
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Rules of engagement

Your envelope is small, so follow these rules:

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- Use approximations where possible, to simplify things.
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Practice question - timescales

The standard S. I. unit for time is the second, but many geological processes deal with longer timescales:

- thousands (× 10^3) of years (ka or kyr)
- millions (× 10^6) of years (Ma or Myr)
- billions (× 10^9) of years (Ga or Byr)

Work out simple approximations for the following, in seconds:

(c) 1 day   (b) 1 week   (c) 1 year

Useful data

- The width of the north Atlantic ocean on a world map accounts for about ¼ of the Earth’s circumference at the latitude of the UK.
- The radius of the Earth is about 6,400 km.
- For an alternative approach, the flight time from the UK to New York, following a small circle above the Atlantic, is about 6 hours. Modern aircraft fly at about 1000 km/hr.

www.oxfordsparks.net/volcano
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Rules of engagement

Your envelope is small, so follow these rules:

- Use scientific notation (× 10ⁿ) to represent numbers.
- Never carry more than 2 significant figures across.
- Use approximations where possible, to simplify things.
- Be confident in estimating typical time- and length-scales. You’ll soon get a sense of whether you are close.

Practice question - timescales

The standard S. I. unit for time is the second, but many geological processes deal with longer timescales:

- thousands (× 10³) of years (ka or kyr)
- millions (× 10⁶) of years (Ma or Myr)
- billions (× 10⁹) of years (Ga or Byr)

Work out simple approximations for the following, in seconds:

(d) 1 day  (b) 1 week  (c) 1 year

What to do

Think about your question (problem D, below). Try to solve it using quantitative reasoning – use the back of a real envelope if you like! If you get stuck, ask your teacher for a solution sheet. Work through the given answer until you are sure you understand the method used. Then try using a slightly different approach to answer the same question.

Next plan how to present your problem to the rest of your group. You will need to consider the questions below:

- What clues will you give if they get stuck?
- Can you think of a similar problem, to give them more practice using the technique in a similar context?
- How will you go through your solution to the problem if anyone is completely stuck?

Your question (problem D)

What is the current rate at which ocean crust is forming as a result of magma rising from below the sea-floor, and what is the ‘typical lifetime’ of the ocean floor?

Useful data

- The oceanic crust is an average of 7 km thick along the 50,000 km of the oceanic spreading ridges.
- The average ‘half-spreading’ rate (increase in sea-floor width on either side of the ridge per year) is 25 mm/yr.
- The oceanic crust covers about ⅔ of the Earth’s surface.
- The radius of the Earth is about 6,400 km.
Problem A – possible solution

In the subduction zone animation we see Ossie speeding up time, to travel along the descending slab, until he reaches the point beneath Stromboli where the fluid escapes from the slab and starts to rise.

In a ‘typical’ subduction zone the depth to the top of the slab is about 100 km. A typical convergence rate in a subduction zone is 5 cm/yr (50 km/Ma). A typical slab dip is 45 degrees.

Distance travelled along the slab top:

\[
\text{distance} = \text{depth} / \cos(45^\circ)
\]
\[
= 100 \text{ km} / 0.7
\]
\[
= 140 \text{ km}
\]

Time taken = distance / speed
\[
= 140 \text{ km} / 5 \times 10^{-5} \text{ km/yr}
\]
\[
= 3 \times 10^6 \text{ yr}
\]
\[
= 3 \text{ Ma}
\]

Comment

Beneath Stromboli, the time could be greater: not only is the distance greater (more like 200 km), but the current rate of plate convergence is lower (currently less than 1 cm/yr), but this is thought to have changed in the past few million years.

We have only calculated the time to go from the top of the trench to the place below the volcano where melting starts. It may well have taken many tens of millions of years for the plate to have travelled from the spreading ridge (where it formed) to the trench, slowly being buried under sediment as it did so.

This means that changes in the nature of the material going down into a subduction zone can take millions of years to feed into the mantle, and then into the volcanoes above. So the signatures of man-made pollution on the seafloor will start to be recycled through subduction zones in the next 2 – 5 Myr.
Problem B – possible solution

The answer to this question is not well known at all! We assume that it is geologically quite fast, but it is very hard to measure. If magma rises through the mantle in blobs (as in a lava lamp), then the rise rate could be as high as a few mm per year; but once the magma starts moving up through cracks, it could move as quickly as several mm/second. During an eruption, magma will travel the last few kilometres up the conduit at great speed, between 1 and 10 m/s.

As a thought experiment, let us assume the following:

- Magma rises 100 km from above the subducting plate at 5 mm/yr.
- The magma then rises 80 km in the cracks of the overlying plate at 1 mm/second.
- In an eruption magma rises 10 km through a conduit at 1 m/second.

How long does this take, ignoring periods of time when the magma sits in pools on the way?

### Rising through the mantle

\[
time = \frac{distance}{speed}
\]

\[
\approx \frac{100 \text{ km}}{5 \times 10^{-5} \text{ km/yr}}
\]

\[
\approx 2 \times 10^6 \text{ yr}
\]

\[
= 2 \text{ Myr}
\]

### Rising in cracks

\[
time = \frac{distance}{speed}
\]

\[
\approx \frac{80 \text{ km}}{1 \times 10^{-5} \text{ km/s}}
\]

\[
= 8 \times 10^6 \text{ s}
\]

\[
\approx \frac{1}{4} \text{ yr}
\]

### Erupting

\[
time = \frac{distance}{speed}
\]

\[
\approx \frac{10 \text{ km}}{1 \times 10^{-3} \text{ km/s}}
\]

\[
= 10^4 \text{ s}
\]

\[
\approx 3 \text{ hr}
\]

**Comment**

Geological evidence suggests that some magmas spend many thousands of years in the crust before eruption; our back of the envelope calculations show that it is possible for magma to escape out of the crust quickly, during eruptions.
**Solution C**

**Solution 1**

The width of the north Atlantic ocean:
On a world map this accounts for about \(\frac{1}{4}\) of the circumference of the Earth at about the latitude of the UK.

Earth’s radius is 6400 km; the southern UK is about 45\(^\circ\) north.

The radius of the small circle at 45\(^\circ\) N = \(\sin(45\,^\circ) \approx 0.7\) of that at the equator.
The radius of the Earth at the equator = 6400 km.
So the radius of the small circle at 45\(^\circ\) N = 0.7 \times 6400 km
= 4500 km

Width of the north Atlantic at 45\(^\circ\) N = \(\frac{1}{4}\) \(\times\) \((2\pi)\times4500\) km
= \(\pi\)/2 \times 4500 km

\(\approx\) 7000 km.

Time for north Atlantic to open = distance / speed
= 7000 km / (2.5 \times 10^{-5}) km/year
= 2.5 \times 10^8 year
= 280 Myr

**Comment:**
Both answers seem reasonable.

However, the main phase of central/north Atlantic opening coincides with the eruption of the Central Atlantic Magmatic Province (CAMP), which is very well exposed in both New England (USA) and Morocco. This event is dated at about 200 Ma, so the long term average rate of plate motion must be closer to 3 cm/yr.

**Solution 2**

The typical flight time from UK to New York, following a small circle across the North Atlantic, is about 6 hours. Modern aircraft fly about 1000 km/hr.

distance = speed \times time
= 1000 km/hr \times 6 hr
= 6000 km

Time for north Atlantic to open = distance / speed
= 6000 km / (2.5 \times 10^{-5}) km/year
= 2.4 \times 10^8 year
= 240 Myr
Problem D – possible solution

What is the current rate at which ocean crust is forming as a result of magma rising from below the sea-floor?

The oceanic crust is an average of 7 km thick along the 50 000 km of the oceanic spreading ridges.

The average ‘half-spreading’ rate is 25 mm/yr, symmetrical about the ridge.

Average rate of new crust formation

= \( (2 \times \text{half spreading rate}) \times (\text{volume}) \)

= \( (2 \times 25 \times 10^{-6}) \text{ km/yr} \times (7 \times 5 \times 10^4) \text{ km}^3 \)

= 18 \text{ km}^3/\text{yr}

What is the ‘typical lifetime’ of the ocean floor?

Oceanic crust covers about \( \frac{3}{4} \) of Earth’s surface.

The mean age of this surface can be estimated as follows:

mean age = area / rate of production of area

area = \( \frac{3}{4} \times 4\pi \times (6400 \text{ km})^2 \)

= \( 8 \times 40 \times 10^6 \text{ km}^2 \)

= \( 3.2 \times 10^8 \text{ km}^2 \)

current rate of production of area = \( 2 \times 5 \times 10^4 \times 25 \times 10^{-6} \)

= \( 2.5 \text{ km}^2/\text{yr} \)

mean age = area / rate of production of area

= \( (3.2 \times 10^8) \text{ km}^2/(2.5) \text{ km}^2/\text{yr} \)

\( \approx 1.3 \times 10^8 \text{ yr} \)

\( \approx 130 \text{ Ma.} \)