Let me get this straight

Mathematical equations can be used to model the spread, growth or decline of information on the internet. It is not always straightforward, however, to look at a set of data and determine whether it fits a particular equation or not. It is much clearer to determine whether data points fit a linear relationship, which produces a straight line graph, than a non-linear one. Often it is useful to transform the variables that we plot in order to generate a straight line graph.

Example

You believe that the spread of information from a source is governed by the equation \( p = 2t^2 + 5 \) where your variables are \( p \), the number of posts, and \( t \) the time since the initial post. You have some data (shown below).

If you plot \( p \) against \( t \), the graph is a curve as expected.

However it is not straightforward to determine whether it fits the equation \( p = 2t^2 + 5 \) or not.

If we replace \( t^2 \) with the letter \( s \) then we could write the equation as: \( p = 2s + 5 \).

This is the same form as the linear equation \( y = mx + c \) where \( m \) is the gradient of a straight line graph and \( y \) is the intercept.
So if we plot $p$ against $t^2$, if the data fits we should get a straight line with a gradient of 2 and an intercept of 5 – which we do.

Your task

If you wanted to test the following relationships between the number of posts, $p$ and the time, $t$, since some event what would you plot in order to get a linear graph? What would be the gradient, and intercept in each case.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Plot $?$ (vertical)</th>
<th>Against $?$ (horizontal)</th>
<th>Gradient</th>
<th>Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p = 2t^2 + 5$</td>
<td>$p$</td>
<td>$t^2$</td>
<td>2</td>
<td>+5</td>
</tr>
<tr>
<td>$p = 3 + \frac{18}{t}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p^2 = 100 + 18t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p = 5at^2 + b^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

You have measured some growth data that you expect follows the equation $p = at^2 + b$. Plot a graph of $p$ against $t^2$ in order to determine the value of $a$ and $b$. 

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>4</td>
<td>4.5</td>
<td>6</td>
<td>8.5</td>
<td>12</td>
<td>16.5</td>
</tr>
<tr>
<td>$t^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Gradient = \[ a = \]
Intercept = \[ b = \]
**Logarithmic Graphs**

If data fits a relationship where \( y = Ax^n \) where \( n \) is a power that you wish to determine. This can be done by taking the log of both sides.

The equation then becomes \( \log(y) = \log(A) + n\log(x) \)

So a plot of \( \log(y) \) against \( \log(x) \) will be a give a straight line of gradient \( n \).

What gradient would you get if you plotted \( \log(p) \) against \( \log(t) \) for the following relationships:

1) \( p = 5t^{1.7} \)
2) \( p = 5t^{1.6} \)
3) \( p = 5t^{2.3} \)

**Natural Logarithms**

If data fits a relationship where \( y = ae^{bx} \) then it can be useful to use natural logarithms (\( \log_e \) or \( \ln \)) since:

If \( y = ae^{bx} \) then \( \ln(y) = \ln(a) + bx \ln(e) \) as \( \ln(e) = 1 \) so that \( \ln(y) = \ln(a) + bx \)

So a graph of \( \ln(y) \) vs \( x \) produces a straight line graph with gradient \( b \) and intercept \( \ln(a) \).

**Please retweet**

The graph shown below is a possible growth chart for the total number of people reached by a particular tweet. The number grows slowly at first, then more rapidly as more and more people retweet it and then finally slows again as most people in the network have seen it.

The equation for this graph is \( y = \frac{200000}{(1+250e^{-kt})} \).

where \( y \) = number of people who have seen the tweet and \( t = \) the number of days since the original tweet

By plotting \( \ln\left(\frac{200000}{y} - 1\right) \) against \( t \) you obtain the following graph with gradient -\( k \). Use the graph below to calculate the value \( k \).

http://www.oxfordsparks.ox.ac.uk/content/keeping-social-media-social